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STRUCTURED CONDITION NUMBERS AND BACKWARD ERRORS IN SCALAR PRODUCT SPACES*

FRANÇOISE TISSEUR† AND STEF GRAILLAT‡

Abstract. The effect of structure-preserving perturbations on the solution to a linear system, matrix inversion, and distance to singularity is investigated. Particular attention is paid to linear and nonlinear structures that form Lie algebras, Jordan algebras and automorphism groups of a scalar product. These include complex symmetric, pseudo-symmetric, persymmetric, skew-symmetric, Hamiltonian, unitary, complex orthogonal and symplectic matrices. Under reasonable assumptions on the scalar product, it is shown that there is little or no difference between structured and unstructured condition numbers and distance to singularity for matrices in Lie and Jordan algebras. Hence, for these classes of matrices, the usual unstructured perturbation analysis is sufficient. It is shown that this is not true in general for structures in automorphism groups. Bounds and computable expressions for the structured condition numbers for a linear system and matrix inversion are derived for these nonlinear structures.

Structured backward errors for the approximate solution of linear systems are also considered. Conditions are given for the structured backward error to be finite. For Lie and Jordan algebras it is proved that, whenever the structured backward error is finite, it is within a small factor of or equal to the unstructured one. The same conclusion holds for orthogonal and unitary structures but cannot easily be extended to other matrix groups.

This work extends and unifies earlier analyses.

Key words. Structured matrices, Normwise structured perturbations, Structured linear systems, Condition number, Backward error, Distance to singularity, Lie algebra, Jordan algebra, Automorphism group, Scalar product, Bilinear form, Sesquilinear form, Orthosymmetric.

AMS subject classifications. 15A12, 65F35, 65F15.

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†School of Mathematics, The University of Manchester, Sackville Street, Manchester, M60 1QD, UK (`ftisseur@ma.man.ac.uk`, `http://www.ma.man.ac.uk/~ftisseur/`). This work was supported by Engineering and Physical Sciences Research Council grant GR/S31693.

‡Laboratoire LP2A, Université de Perpignan Via Domitia, 52, avenue Paul Alduy, F-66860 Perpignan Cedex, France (`graillat@univ-perp.fr`, `http://gala.univ-perp.fr/~graillat/`).