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FIBONACCI-HORNER DECOMPOSITION OF THE MATRIX EXPONENTIAL AND THE FUNDAMENTAL SYSTEM OF SOLUTIONS*

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Abstract. This paper concerns the Fibonacci-Horner decomposition of the matrix powers A^n and the matrix exponential e^{tA} ($A \in M(r;\mathbb{C})$, $t \in \mathbb{R}$), which is derived from the combinatorial properties of the generalized Fibonacci sequences in the algebra of square matrices. More precisely, e^{tA} is expressed in a natural way in the so–called Fibonacci-Horner basis with the aid of the dynamical solution of the associated ordinary differential equation. Two simple processes for computing the dynamical solution and the fundamental system of solutions are given. The connection to Verde-Star's approach is discussed. Moreover, an extension to the computation of $f(A)$, where f is an analytic function is initiated. Finally, some illustrative examples are presented.

Key words. Matrix powers, Matrix exponential, Dynamical solution, Generalized Fibonacci sequence, Fibonacci-Horner decomposition, Matrix functions.

AMS subject classifications. 15A18, 15A99.

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