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SEMITRANSITIVE SUBSPACES OF MATRICES*

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Abstract. A set of matrices $\mathcal{S} \subseteq \mathbb{M}_n(\mathbb{F})$ is said to be semitransitive if for any two nonzero vectors $x, y \in \mathbb{F}^n$, there exists a matrix $A \in \mathcal{S}$ such that either $Ax = y$ or $Ay = x$. In this paper various properties of semitransitive linear subspaces of $\mathbb{M}_n(\mathbb{F})$ are studied. In particular, it is shown that every semitransitive subspace of matrices has a cyclic vector. Moreover, if $|\mathbb{F}| \geq n$, it always contains an invertible matrix. It is proved that there are minimal semitransitive matrix spaces without any nontrivial invariant subspace. The structure of minimal semitransitive spaces and triangularizable semitransitive spaces is also studied. Among other results it is shown that every triangularizable semitransitive subspace contains a nonzero nilpotent.

Key words. Semitransitive subspaces of matrices, Triangularizability, Reducibility, Minimality.

AMS subject classifications. 15A30.

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