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ZEROS OF UNILATERAL QUATERNIONIC POLYNOMIALS*

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Abstract. The purpose of this paper is to show how the problem of finding the zeros of unilateral n-order quaternionic polynomials can be solved by determining the eigenvectors of the corresponding companion matrix. This approach, probably superfluous in the case of quadratic equations for which a closed formula can be given, becomes truly useful for (unilateral) n-order polynomials. To understand the strength of this method, it is compared with the Niven algorithm and it is shown where this (full) matrix approach improves previous methods based on the use of the Niven algorithm. For convenience of the readers, some examples of second and third order unilateral quaternionic polynomials are explicitly solved. The leading idea of the practical solution method proposed in this work can be summarized in the following three steps: translating the quaternionic polynomial in the eigenvalue problem for its companion matrix, finding its eigenvectors, and, finally, giving the quaternionic solution of the unilateral polynomial in terms of the components of such eigenvectors. A brief discussion on bilateral quaternionic quadratic equations is also presented.

Key words. Quaternions, Eigenvalue problem, Matrices over special rings.

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