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LINEAR COMBINATIONS OF GRAPH EIGENVALUES*

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Abstract. Let $\mu_1(G) \geq \ldots \geq \mu_n(G)$ be the eigenvalues of the adjacency matrix of a graph G of order n, and $\overline{G}$ be the complement of G. Suppose $F(G)$ is a fixed linear combination of $\mu_i(G)$, $\mu_{n-i+1}(G)$, $\mu_i\left(\overline{G}\right)$, and $\mu_{n-i+1}\left(\overline{G}\right)$, $1 \leq i \leq k$. It is shown that the limit

$$\lim_{n\to\infty} \frac{1}{n} \max\{F(G) : v(G) = n\}$$

always exists. Moreover, the statement remains true if the maximum is taken over some restricted families like "K_r-free" or "r-partite" graphs. It is also shown that

$$\frac{29+\sqrt{329}}{42}n - 25 \leq \max_{v(G)=n} \mu_1(G) + \mu_2(G) \leq \frac{2}{\sqrt{3}}n.$$

This inequality answers in the negative a question of Gernert.

Key words. Extremal graph eigenvalues, Linear combination of eigenvalues, Multiplicative property.

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