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ON CLASSIFICATION OF NORMAL MATRICES IN INDEFINITE INNER PRODUCT SPACES*

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Abstract. Canonical forms are developed for several sets of matrices that are normal with respect to an indefinite inner product induced by a nonsingular Hermitian, symmetric, or skew-symmetric matrix. The most general result covers the case of polynomially normal matrices, i.e., matrices whose adjoint with respect to the indefinite inner product is a polynomial of the original matrix. From this result, canonical forms for complex matrices that are selfadjoint, skewadjoint, or unitary with respect to the given indefinite inner product are derived. Most of the canonical forms for the latter three special types of normal matrices are known in the literature, but it is the aim of this paper to present a general theory that allows the unified treatment of all different cases and to collect known results and new results such that all canonical forms for the complex case can be found in a single source.

Key words. Indefinite inner products, Sesquilinear forms, Bilinear forms, Normal matrices, Selfadjoint matrices, Skewadjoint matrices, Unitary matrices.

AMS subject classifications. 15A21, 15A57, 15A63.

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