Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 15, pp. 84-106, February 2006

ELA
http://math.technion.ac.il/iic/ela

ESSENTIAL DECOMPOSITION OF POLYNOMIALLY NORMAL MATRICES IN REAL INDEFINITE INNER PRODUCT SPACES*

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Abstract. Polynomially normal matrices in real indefinite inner product spaces are studied, i.e., matrices whose adjoint with respect to the indefinite inner product is a polynomial in the matrix. The set of these matrices is a subset of indefinite inner product normal matrices that contains all selfadjoint, skew-adjoint, and unitary matrices, but that is small enough such that all elements can be completely classified. The essential decomposition of a real polynomially normal matrix is introduced. This is a decomposition into three parts, one part having real spectrum only and two parts that can be described by two complex matrices that are polynomially normal with respect to a sesquilinear and bilinear form, respectively. In the paper, the essential decomposition is used as a tool in order to derive a sufficient condition for existence of invariant semidefinite subspaces and to obtain canonical forms for real polynomially normal matrices. In particular, canonical forms for real matrices that are selfadjoint, skewadjoint, or unitary with respect to an indefinite inner product are recovered.

Key words. Indefinite inner products, Normal matrices, Selfadjoint matrices, Skewadjoint matrices, Unitary matrices, Essential decomposition.

AMS subject classifications. 15A21, 15A57, 15A63.

*Received by the editors on 12 August 2005. Accepted for publication on 2 February 2006. Handling Editor: Peter Lancaster.

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