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NOTE ON DELETING A VERTEX AND WEAK INTERLACING OF THE LAPLACIAN SPECTRUM*

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Abstract. The question of what happens to the eigenvalues of the Laplacian of a graph when we delete a vertex is addressed. It is shown that

$$\lambda_i - 1 \leq \lambda_i^v \leq \lambda_{i+1},$$

where λ_i is the ith smallest eigenvalues of the Laplacian of the original graph and λ_i^v is the ith smallest eigenvalues of the Laplacian of the graph $G[V-v]$; i.e., the graph obtained after removing the vertex v. It is shown that the average number of leaves in a random spanning tree $\mathcal{F}(G) > \frac{2|E|e^{\frac{-1}{\alpha}}}{\lambda_n}$, if $\lambda_2 > \alpha n$.

Key words. Spectrum, Random spanning trees, Cayley formula, Laplacian, Number of leaves.

AMS subject classifications. 05C30, 34L15, 34L40.

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