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A GENERALIZATION OF ROTATIONS AND HYPERBOLIC MATRICES AND ITS APPLICATIONS*

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Abstract. In this paper, A-factor circulant matrices with the structure of a circulant, but with the entries below the diagonal multiplied by the same factor A are introduced. Then the generalized rotation and hyperbolic matrices are defined, using an idea due to Ungar. Considering the exponential property of the generalized rotation and hyperbolic matrices, additive formulae for corresponding matrices are also obtained. Also introduced is the block Fourier matrix as a basis for generalizing the Euler formula. The special functions associated with the corresponding Lie group are the functions $F^A_{n,k}(x)$ $(k = 0, 1, \cdots, n-1)$. As an application, the fundamental solutions of the second order matrix differential equation $y''(x) = \Pi_A y(x)$ with initial conditions $y(0) = I$ and $y'(0) = 0$ are obtained using the generalized trigonometric functions $\cos_A(x)$ and $\sin_A(x)$.

Key words. Circulant matrix, A-Factor circulant matrices, Block Vandermonde and Fourier matrices, Rotation and hyperbolic matrices, Generalized Euler formula matrices, Periodic solutions.

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