Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 16, pp. 157-161, July 2007

ELA
http://math.technion.ac.il/iic/ela

ON PERFECT CONDITIONING OF VANDERMONDE MATRICES ON THE UNIT CIRCLE*

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Abstract. Let $K, M \in \mathbb{N}$ with $K < M$, and define a square $K \times K$ Vandermonde matrix $A = A\left(\tau, \overrightarrow{n}\right)$ with nodes on the unit circle: $A_{p,q} = \exp\left(-j2\pi p n_q \tau / K\right); p, q = 0, 1, ..., K-1$, where $n_q \in \{0, 1, ..., M-1\}$ and $n_0 < n_1 < .... < n_{K-1}$. Such matrices arise in some types of interpolation problems. In this paper, necessary and sufficient conditions are presented on the vector $\overrightarrow{n}$ so that a value of $\tau \in \mathbb{R}$ can be found to achieve perfect conditioning of A. A simple test to check the condition is derived and the corresponding value of τ is found.

Key words. Vandermonde matrix, Condition number, Perfect conditioning.

AMS subject classifications. 15A12, 65F35.

*Received by the editors 3 November 2006. Accepted for publication 8 July 2007. Handling Editor: Michael Neumann.

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