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THE MOORE-PENROSE INVERSE OF A FREE MATRIX*

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Abstract. A matrix is free, or generic, if its nonzero entries are algebraically independent. Necessary and sufficient combinatorial conditions are presented for a complex free matrix to have a free Moore-Penrose inverse. These conditions extend previously known results for square, nonsingular free matrices. The result used to prove this characterization relates the combinatorial structure of a free matrix to that of its Moore-Penrose inverse. Also, it is proved that the bipartite graph or, equivalently, the zero pattern of a free matrix uniquely determines that of its Moore-Penrose inverse, and this mapping is described explicitly. Finally, it is proved that a free matrix contains at most as many nonzero entries as does its Moore-Penrose inverse.

Key words. Free matrix, Moore-Penrose inverse, Bipartite graph, Directed graph.

AMS subject classifications. 05C50, 15A09.

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