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A RELATION BETWEEN THE MULTIPLICITY OF THE SECOND EIGENVALUE OF A GRAPH LAPLACIAN, COURANT'S NODAL LINE THEOREM AND THE SUBSTANTIAL DIMENSION OF TIGHT POLYHEDRAL SURFACES*

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Abstract. A relation between the multiplicity m of the second eigenvalue λ_2 of a Laplacian on a graph G, tight mappings of G and a discrete analogue of Courant's nodal line theorem is discussed. For a certain class of graphs, it is shown that the m-dimensional eigenspace of λ_2 is tight and thus defines a tight mapping of G into an m-dimensional Euclidean space. The tightness of the mapping is shown to set Colin de Verdiére's upper bound on the maximal λ_2-multiplicity, $m \leq chr(\gamma(G)) - 1$, where $chr(\gamma(G))$ is the chromatic number and $\gamma(G)$ is the genus of G.

Key words. Graph Laplacian, Tight Embedding, Nodal domains, Eigenfunctions, Polyhedral Manifolds.

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