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AFFINE TRANSFORMATIONS OF A LEONARD PAIR*

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Abstract. Let $\mathbb{K}$ denote a field and let V denote a vector space over $\mathbb{K}$ with finite positive dimension. An ordered pair is considered of linear transformations $A : V \to V$ and $A^* : V \to V$ that satisfy (i) and (ii) below:

(i) There exists a basis for V with respect to which the matrix representing A is irreducible tridiagonal and the matrix representing A^* is diagonal.
(ii) There exists a basis for V with respect to which the matrix representing A^* is irreducible tridiagonal and the matrix representing A is diagonal.

Such a pair is called a *Leonard pair* on V. Let $\xi, \zeta, \xi^*, \zeta^*$ denote scalars in $\mathbb{K}$ with ξ, ξ^* nonzero, and note that $\xi A + \zeta I$, $\xi^* A^* + \zeta^* I$ is a Leonard pair on V. Necessary and sufficient conditions are given for this Leonard pair to be isomorphic to A, A^*. Also given are necessary and sufficient conditions for this Leonard pair to be isomorphic to the Leonard pair A^*, A.

Key words. Leonard pair, Tridiagonal pair, q-Racah polynomial, Orthogonal polynomial.

AMS subject classifications. 05E35, 05E30, 33C45, 33D45.

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