

ON SOLUTIONS TO THE QUATERNION MATRIX EQUATION

$$AXB + CYD = E^*$$

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Abstract. Expressions, as well as necessary and sufficient conditions are given for the existence of the real and pure imaginary solutions to the consistent quaternion matrix equation $AXB + CYD = E$. Formulas are established for the extreme ranks of real matrices $X_i, Y_i, i = 1, \dots, 4$, in a solution pair $X = X_1 + X_2i + X_3j + X_4k$ and $Y = Y_1 + Y_2i + Y_3j + Y_4k$ to this equation. Moreover, necessary and sufficient conditions are derived for all solution pairs X and Y of this equation to be real or pure imaginary, respectively. Some known results can be regarded as special cases of the results in this paper.

Key words. Quaternion matrix equation, Extreme rank, Generalized inverse.

AMS subject classifications. 15A03, 15A09, 15A24, 15A33.

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