

THE ANTI-SYMMETRIC ORTHO-SYMMETRIC SOLUTIONS OF THE MATRIX EQUATION $A^T X A = D^*$

QING-FENG XIAO[†], XI-YAN HU[†], AND LEI ZHANG[†]

Abstract. In this paper, the following problems are discussed.

Problem I. Given matrices $A \in R^{n \times m}$ and $D \in R^{m \times m}$, find $X \in ASR_P^n$ such that $A^T X A = D$, where

$$ASR_P^n = \{X \in ASR^{n \times n} | P X \in SR^{n \times n} \text{ for given } P \in OR^{n \times n} \text{ satisfying } P^T = P\}.$$

Problem II. Given a matrix $\tilde{X} \in R^{n \times n}$, find $\hat{X} \in S_E$ such that

$$\|\tilde{X} - \hat{X}\| = \inf_{X \in S_E} \|\tilde{X} - X\|,$$

where $\|\cdot\|$ is the Frobenius norm, and S_E is the solution set of Problem I.

Expressions for the general solution of Problem I are derived. Necessary and sufficient conditions for the solvability of Problem I are provided. For Problem II, an expression for the solution is given as well.

Key words. Anti-symmetric ortho-symmetric matrix, Matrix equation, Matrix nearness problem, Optimal approximation, Least-square solutions.

AMS subject classifications. 65F15, 65F20.

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[†]College of Mathematics and Econometrics, Hunan University, Changsha 410082, P.R. of China (qfxiao@hnu.cn). Research supported by National Natural Science Foundation of China (under Grant 10571047) and the Doctorate Foundation of the Ministry of Education of China (under Grant 20060532014).