

## MATRICES TOTALLY POSITIVE RELATIVE TO A TREE\*

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**Abstract.** It is known that for a totally positive (TP) matrix, the eigenvalues are positive and distinct and the eigenvector associated with the smallest eigenvalue is totally nonzero and has an alternating sign pattern. Here, a certain weakening of the TP hypothesis is shown to yield a similar conclusion.

**Key words.** Totally positive matrices, Sylvester's identity, Graph theory, Spectral theory.

**AMS subject classifications.** 15A18, 94C15.

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