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LINEARIZATIONS OF SINGULAR MATRIX POLYNOMIALS AND THE RECOVERY OF MINIMAL INDICES*

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Abstract. A standard way of dealing with a regular matrix polynomial $P(\lambda)$ is to convert it into an equivalent matrix pencil – a process known as linearization. Two vector spaces of pencils $\mathbb{L}_1(P)$ and $\mathbb{L}_2(P)$ that generalize the first and second companion forms have recently been introduced by Mackey, Mackey, Mehl and Mehrmann. Almost all of these pencils are linearizations for $P(\lambda)$ when P is regular. The goal of this work is to show that most of the pencils in $\mathbb{L}_1(P)$ and $\mathbb{L}_2(P)$ are still linearizations when $P(\lambda)$ is a singular square matrix polynomial, and that these linearizations can be used to obtain the complete eigenstructure of $P(\lambda)$, comprised not only of the finite and infinite eigenvalues, but also for singular polynomials of the left and right minimal indices and minimal bases. We show explicitly how to recover the minimal indices and bases of the polynomial $P(\lambda)$ from the minimal indices and bases of linearizations in $\mathbb{L}_1(P)$ and $\mathbb{L}_2(P)$. As a consequence of the recovery formulae for minimal indices, we prove that the vector space $\mathbb{DL}(P) = \mathbb{L}_1(P) \cap \mathbb{L}_2(P)$ will never contain any linearization for a square singular polynomial $P(\lambda)$. Finally, the results are extended to other linearizations of singular polynomials defined in terms of more general polynomial bases.

Key words. Singular matrix polynomials, Matrix pencils, Minimal indices, Minimal bases, Linearization.

AMS subject classifications. 15A18, 15A21, 15A22, 65F15.

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