

## UNIVERSALLY OPTIMAL MATRICES AND FIELD INDEPENDENCE OF THE MINIMUM RANK OF A GRAPH\*

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**Abstract.** The minimum rank of a simple graph  $G$  over a field  $F$  is the smallest possible rank among all symmetric matrices over  $F$  whose  $(i, j)$ th entry (for  $i \neq j$ ) is nonzero whenever  $\{i, j\}$  is an edge in  $G$  and is zero otherwise. A universally optimal matrix is defined to be an integer matrix  $A$  such that every off-diagonal entry of  $A$  is 0, 1, or  $-1$ , and for all fields  $F$ , the rank of  $A$  is the minimum rank over  $F$  of its graph. Universally optimal matrices are used to establish field independence of minimum rank for numerous graphs. Examples are also provided verifying lack of field independence for other graphs.

**Key words.** Minimum rank, Universally optimal matrix, Field independent, Symmetric matrix, Rank, Graph, Matrix.

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