

GENERALIZED PASCAL TRIANGLES AND TOEPLITZ MATRICES*

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Abstract. The purpose of this article is to study determinants of matrices which are known as generalized Pascal triangles (see R. Bacher. Determinants of matrices related to the Pascal triangle. *J. Théor. Nombres Bordeaux*, 14:19–41, 2002). This article presents a factorization by expressing such a matrix as a product of a unipotent lower triangular matrix, a Toeplitz matrix, and a unipotent upper triangular matrix. The determinant of a generalized Pascal matrix equals thus the determinant of a Toeplitz matrix. This equality allows for the evaluation of a few determinants of generalized Pascal matrices associated with certain sequences. In particular, families of quasi-Pascal matrices are obtained whose leading principal minors generate any arbitrary linear subsequences $(\mathcal{F}_{nr+s})_{n \geq 1}$ or $(\mathcal{L}_{nr+s})_{n \geq 1}$ of the Fibonacci or Lucas sequence. New matrices are constructed whose entries are given by certain linear non-homogeneous recurrence relations, and the leading principal minors of which form the Fibonacci sequence.

Key words. Determinant, Matrix factorization, Generalized Pascal triangle, Generalized symmetric (skymmetric) Pascal triangle, Toeplitz matrix, Recursive relation, Fibonacci (Lucas, Catalan) sequence, Golden ratio.

AMS subject classifications. 15A15, 11C20.

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