

ZERO-NONZERO PATTERNS FOR NILPOTENT MATRICES OVER FINITE FIELDS*

KEVIN N. VANDER MEULEN[†] AND ADAM VAN TUYL[‡]

Abstract. Fix a field \mathbb{F} . A zero-nonzero pattern \mathcal{A} is said to be potentially nilpotent over \mathbb{F} if there exists a matrix with entries in \mathbb{F} with zero-nonzero pattern \mathcal{A} that allows nilpotence. In this paper an investigation is initiated into which zero-nonzero patterns are potentially nilpotent over \mathbb{F} with a special emphasis on the case that $\mathbb{F} = \mathbb{Z}_p$ is a finite field. A necessary condition on \mathbb{F} is observed for a pattern to be potentially nilpotent when the associated digraph has m loops but no small k -cycles, $2 \leq k \leq m - 1$. As part of this investigation, methods are developed, using the tools of algebraic geometry and commutative algebra, to eliminate zero-nonzero patterns \mathcal{A} as being potentially nilpotent over any field \mathbb{F} . These techniques are then used to classify all irreducible zero-nonzero patterns of order two and three that are potentially nilpotent over \mathbb{Z}_p for each prime p .

Key words. Zero-nonzero patterns, Nilpotent, Ideal saturation, Gröbner basis, Finite fields.

AMS subject classifications. 15A18, 13P10, 05C50, 11T06.

* Received by the editors December 18, 2008. Accepted for publication October 8, 2009. Handling Editor: Michael J. Tsatsomeros.

[†]Department of Mathematics, Redeemer University College, Ancaster, ON L9K 1J4 Canada (kvander@redeemer.ca). Research supported in part by an NSERC Discovery Grant.

[‡]Department of Mathematical Sciences, Lakehead University, Thunder Bay, ON P7B 5E1 Canada (avantuy@lakeheadu.ca). Research supported in part by an NSERC Discovery Grant.