

SPECTRALLY ARBITRARY COMPLEX SIGN PATTERN MATRICES*

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Abstract. An $n \times n$ complex sign pattern matrix \mathcal{S} is said to be spectrally arbitrary if for every monic n th degree polynomial $f(\lambda)$ with coefficients from \mathbb{C} , there is a complex matrix in the complex sign pattern class of \mathcal{S} such that its characteristic polynomial is $f(\lambda)$. If \mathcal{S} is a spectrally arbitrary complex sign pattern matrix, and no proper subpattern of \mathcal{S} is spectrally arbitrary, then \mathcal{S} is a minimal spectrally arbitrary complex sign pattern matrix. This paper extends the Nilpotent-Jacobian method for sign pattern matrices to complex sign pattern matrices, establishing a means to show that an irreducible complex sign pattern matrix and all its superpatterns are spectrally arbitrary. This method is then applied to prove that for every $n \geq 2$ there exists an $n \times n$ irreducible, spectrally arbitrary complex sign pattern with exactly $3n$ nonzero entries. In addition, it is shown that every $n \times n$ irreducible, spectrally arbitrary complex sign pattern matrix has at least $3n - 1$ nonzero entries.

Key words. Complex sign pattern, Spectrally arbitrary pattern, Nilpotent.

AMS subject classifications. 15A18, 05C15.

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