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MATRICES OF POSITIVE POLYNOMIALS*

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Abstract. Associated to a square matrix all of whose entries are real Laurent polynomials in several variables with no negative coefficients is an ordered "dimension" module introduced by Tuncel, with additional structure, which acts as an invariant for topological Markov chains, and is also an invariant for actions of tori on AF C*-algebras. In describing this invariant, we are led naturally to eventually positivity questions, which in turn lead to descriptions of the Poisson boundaries (of random walks affiliated with these processes). There is an interplay between the algebraic, dynamical, and probabilistic aspects, for example, if the (suitably defined) endomorphism ring of the dimension module is noetherian, then the boundary is more easily described, the asymptotic behaviour of powers of the matrix is tractible, and the order-theoretic aspects of the dimension module are less difficult to deal with than in general. We also show that under relatively modest conditions, the largest eigenvalue function is a complete invariant for finite equivalence (early results of Marcus and Tuncel showed that it is not a complete invariant in general, but is so if the large eigenvalue is a polynomial).

Key words. positive polynomial, trace, point evaluation, Newton polyhedron, convex polytope, primitive matrix, random walk, order ideal, (topological) shift equivalence, finite equivalence, Choquet simplex, real analytic function.

AMS subject classifications. 37B10, 46B40, 46L80, 32A38, 60J05, 16W80, 19K14, 15A54.

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