

## SIGN PATTERNS THAT ALLOW EVENTUAL POSITIVITY\*

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**Abstract.** Several necessary or sufficient conditions for a sign pattern to allow eventual positivity are established. It is also shown that certain families of sign patterns do not allow eventual positivity. These results are applied to show that for  $n \geq 2$ , the minimum number of positive entries in an  $n \times n$  sign pattern that allows eventual positivity is  $n + 1$ , and to classify all  $2 \times 2$  and  $3 \times 3$  sign patterns as to whether or not the pattern allows eventual positivity. A  $3 \times 3$  matrix is presented to demonstrate that the positive part of an eventually positive matrix need not be primitive, answering negatively a question of Johnson and Tarazaga.

**Key words.** Eventually positive matrix, Potentially eventually positive sign pattern, Perron-Frobenius, Directed graph.

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