

## SIGN PATTERNS THAT REQUIRE EVENTUAL POSITIVITY OR REQUIRE EVENTUAL NONNEGATIVITY\*

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**Abstract.** It is shown that a square sign pattern  $\mathcal{A}$  requires eventual positivity if and only if it is nonnegative and primitive. Let the set of vertices in the digraph of  $\mathcal{A}$  that have access to a vertex  $s$  be denoted by  $\text{In}(s)$  and the set of vertices to which  $t$  has access denoted by  $\text{Out}(t)$ . It is shown that  $\mathcal{A} = [\alpha_{ij}]$  requires eventual nonnegativity if and only if for every  $s, t$  such that  $\alpha_{st} = -$ , the two principal submatrices of  $\mathcal{A}$  indexed by  $\text{In}(s)$  and  $\text{Out}(t)$  require nilpotence. It is shown that  $\mathcal{A}$  requires eventual exponential positivity if and only if it requires exponential positivity, i.e.,  $\mathcal{A}$  is irreducible and its off-diagonal entries are nonnegative.

**Key words.** Eventually nonnegative matrix, Eventually positive matrix, Eventually exponentially positive matrix, Exponentially positive matrix, Sign pattern, Perron-Frobenius.

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