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KANTOROVICH TYPE INEQUALITIES FOR ORDERED LINEAR SPACES*

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Abstract. In this paper Kantorovich type inequalities are derived for linear spaces endowed with bilinear operations $\circ_1$ and $\circ_2$. Sufficient conditions are found for vector-valued maps Φ and Ψ and vectors x and y under which the inequality

$$\Phi(x) \circ_2 \Phi(y) \leq \frac{C+c}{2\sqrt{Cc}}\, \Psi(x \circ_1 y)$$

is satisfied. Complementary inequalities are also given. Some results of Dragomir [J. Inequal. Pure Appl. Math., 5 (3), Art. 76, 2004] and Bourin [Linear Algebra Appl., 416:890–907, 2006] are generalized. The inequalities are applied to C^*-algebras and unital positive maps.

Key words. Kantorovich type inequality, Linear space, Bilinear operation, Preorder, C^*-algebra, Unital positive map, Matrix.

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