

## RANGES OF SYLVESTER MAPS AND A MINIMAL RANK PROBLEM\*

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**Abstract.** It is proved that the range of a Sylvester map defined by two matrices of sizes  $p \times p$  and  $q \times q$ , respectively, plus matrices whose ranks are bounded above, cover all  $p \times q$  matrices. The best possible upper bound on the ranks is found in many cases. An application is made to a minimal rank problem that is motivated by the theory of minimal factorizations of rational matrix functions.

**Key words.** Sylvester maps, Invariant subspaces, Rank.

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