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EXTREMAL ALGEBRAIC CONNECTIVITIES OF CERTAIN CATERPILLAR CLASSES AND SYMMETRIC CATERPILLARS*

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Abstract. A caterpillar is a tree in which the removal of all pendant vertices makes it a path. Let $d \geq 3$ and $n \geq 6$ be given. Let P_{d-1} be the path of $d-1$ vertices and S_p be the star of $p+1$ vertices. Let $\mathbf{p} = [p_1, p_2, ..., p_{d-1}]$ such that $p_1 \geq 1, p_2 \geq 1, ..., p_{d-1} \geq 1$. Let $C(\mathbf{p})$ be the caterpillar obtained from the stars $S_{p_1}, S_{p_2}, ..., S_{p_{d-1}}$ and the path P_{d-1} by identifying the root of S_{p_i} with the $i-$vertex of P_{d-1}. Let $n > 2(d-1)$ be given. Let

$$\mathcal{C} = \{C(\mathbf{p}) : p_1 + p_2 + ... + p_{d-1} = n - d + 1\}$$

and

$$\mathcal{S} = \{C(\mathbf{p}) \in \mathcal{C} : p_j = p_{d-j},\ j = 1, 2, \cdots, \lfloor \frac{d-1}{2} \rfloor\}.$$

In this paper, the caterpillars in $\mathcal{C}$ and in $\mathcal{S}$ having the maximum and the minimum algebraic connectivity are found. Moreover, the algebraic connectivity of a caterpillar in $\mathcal{S}$ as the smallest eigenvalue of a 2×2 - block tridiagonal matrix of order $2s \times 2s$ if $d = 2s + 1$ or $d = 2s + 2$ is characterized.

Key words. Laplacian matrix, Algebraic connectivity, Caterpillar, Bottleneck matrices, Perron branches, Characteristic vertices.

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