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SPECTRALLY ARBITRARY TREE SIGN PATTERNS OF ORDER 4*

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Abstract. A sign pattern matrix (or a sign pattern, or a pattern) is a matrix whose entries are from the set $\{+, -, 0\}$. An $n \times n$ sign pattern matrix is a spectrally arbitrary pattern (SAP) if for every monic real polynomial $p(x)$ of degree n, there exists a real matrix B whose entries agree in sign with A such that the characteristic polynomial of B is $p(x)$. An $n \times n$ sign pattern A is an inertially arbitrary pattern (IAP) if (r, s, t) belongs to the inertia set of A for every nonnegative integer triple (r, s, t) with $r + s + t = n$. Tree sign patterns are investigated, with a special emphasis on 4×4 tridiagonal sign patterns. The set of spectrally arbitrary sign patterns is a subset of the set of potentially stable sign patterns, and for tree sign patterns of order 4, the set of all potentially stable sign patterns is known. The main results are obtained by determining which of these potentially stable sign patterns are SAPs. Necessary and sufficient conditions for an irreducible 4×4 tridiagonal pattern to be an SAP are found. As a result, all 4×4 tree sign patterns that are SAPs are characterized. A new technique, an innovative application of Gröbner bases for demonstrating that a sign pattern is not potentially nilpotent, is introduced. Connections between the SAP classes and the classes of potentially nilpotent and potentially stable patterns are explored. Some interesting open questions are also provided.

Key words. Sign pattern matrix, Spectrally arbitrary pattern, Inertially arbitrary pattern, Tree sign pattern, Potentially nilpotent pattern, Potentially stable pattern, Gröbner basis.

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