

REFINED INERTIALLY AND SPECTRALLY ARBITRARY ZERO-NONZERO PATTERNS*

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Abstract. The refined inertia of a matrix is a quadruple specifying its inertia and additionally the number of its eigenvalues equal to zero. Spectral properties, especially the refined inertias, of real matrices with a given zero-nonzero pattern are investigated. It is shown that every zero-nonzero refined inertially arbitrary pattern of order 4 or less is also spectrally arbitrary. Irreducible and reducible examples are presented to show that for higher orders this is not the case. A further example shows that two zero-nonzero patterns that are not refined inertially arbitrary can have a direct sum that is refined inertially arbitrary, paralleling a known result for inertially arbitrary patterns. Analogously, it is shown that the direct sum of two zero-nonzero patterns may be spectrally arbitrary even if neither summand is spectrally arbitrary.

Key words. Zero-nonzero pattern, Refined inertia, Inertially arbitrary, Spectrally arbitrary.

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