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EFFICIENT COMPUTATION OF ENCLOSURES FOR THE EXACT SOLVENTS OF A QUADRATIC MATRIX EQUATION*

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Abstract. None of the usual floating point numerical techniques available for solving the quadratic matrix equation $AX^2 + BX + C = 0$ with square matrices A, B, C and X, can provide an exact solution; they can just obtain approximations to an exact solution. We use interval arithmetic to compute an interval matrix which contains an exact solution to this quadratic matrix equation, where we aim at obtaining narrow intervals for each entry. We propose a residual version of a modified Krawczyk operator which has a cubic computational complexity, provided that A is nonsingular and X and $X + A^{-1}B$ are diagonalizable. For the case that A is singular or nearly singular, but B is nonsingular we provide an enclosure method analogous to a functional iteration method. Numerical examples have also been given.

Key words. Quadratic matrix equation, Matrix square root, Interval analysis, Krawczyk operator, Automatic result verification.

AMS subject classifications. 65G20, 65F30.

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