

ON THE MINIMUM VECTOR RANK OF MULTIGRAPHS*

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Abstract. The minimum vector rank (mvr) of a graph over a field \mathbb{F} is the smallest d for which a faithful vector representation of G exists in \mathbb{F}^d . For simple graphs, minimum semidefinite rank (msr) and minimum vector rank differ by exactly the number of isolated vertices. We explore the relationship between msr and mvr for multigraphs and show that a result linking the msr of chordal graphs to clique cover number also holds for the mvr of multigraphs. We study the difference between msr and mvr in the removal of duplicate vertices in multigraphs, and relate mvr to certain coloring problems.

Key words. Minimum semidefinite rank, Minimum vector rank, Duplicate vertices, Chordal graph.

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