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MATRIX FUNCTIONS PRESERVING SETS
OF GENERALIZED NONNEGATIVE MATRICES[*]

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Abstract. Matrix functions preserving several sets of generalized nonnegative matrices are characterized. These sets include PFn, the set of $n \times n$ real eventually positive matrices; and WPFn, the set of matrices $A \in \mathbb{R}^{n \times n}$ such that A and its transpose have the Perron-Frobenius property. Necessary conditions and sufficient conditions for a matrix function to preserve the set of $n \times n$ real eventually nonnegative matrices and the set of $n \times n$ real exponentially nonnegative matrices are also presented. In particular, it is shown that if $f(0) \neq 0$ and $f'(0) \neq 0$ for some entire function f, then such an entire function does not preserve the set of $n \times n$ real eventually nonnegative matrices. It is also shown that the only complex polynomials that preserve the set of $n \times n$ real exponentially nonnegative matrices are $p(z) = az + b$, where $a, b \in \mathbb{R}$ and $a \geq 0$.

Key words. Matrix functions, Generalization of nonnegative matrices, Eventually nonnegative matrices, Eventually positive matrices, Exponentially nonnegative matrices, Eventually exponentially nonnegative matrices, Perron-Frobenius property, Strong Perron-Frobenius property.

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