

EUCLIDEAN AND CIRCUM-EUCLIDEAN DISTANCE MATRICES: CHARACTERIZATIONS AND LINEAR PRESERVERS*

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Abstract. Short proofs are given to various characterizations of the (circum-)Euclidean squared distance matrices. Linear preserver problems related to these matrices are discussed.

Key words. Distance geometry, Multidimensional scaling, Eigenvalues, Interlacing inequalities, Linear preservers.

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