

**BOUNDS ON THE SPECTRAL RADIUS OF A  
HADAMARD PRODUCT OF NONNEGATIVE OR  
POSITIVE SEMIDEFINITE MATRICES\***

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**Abstract.** X. Zhan has conjectured that the spectral radius of the Hadamard product of two square nonnegative matrices is not greater than the spectral radius of their ordinary product. We prove Zhan's conjecture, and a related inequality for positive semidefinite matrices, using standard facts about principal submatrices, Kronecker products, and the spectral radius.

**Key words.** Hadamard product, Nonnegative matrix, Positive semidefinite matrix, Positive definite matrix, Spectral radius, Kronecker product, Matrix inequality.

**AMS subject classifications.** 15A45, 15A48, 15A69.

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