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CHARACTERIZATIONS OF JORDAN DERIVATIONS ON TRIANGULAR RINGS: ADDITIVE MAPS JORDAN DERIVABLE AT IDEMPOTENTS*

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Abstract. Let $\mathcal{T}$ be a triangular ring. An additive map δ from $\mathcal{T}$ into itself is said to be Jordan derivable at an element $Z \in \mathcal{T}$ if $\delta(A)B + A\delta(B) + \delta(B)A + B\delta(A) = \delta(AB + BA)$ for any $A, B \in \mathcal{T}$ with $AB + BA = Z$. An element $Z \in \mathcal{T}$ is called a Jordan all-derivable point of $\mathcal{T}$ if every additive map Jordan derivable at Z is a Jordan derivation. In this paper, we show that some idempotents in $\mathcal{T}$ are Jordan all-derivable points. As its application, we get the result that for any nest $\mathcal{N}$ in a factor von Neumann algebra $\mathcal{R}$, every nonzero idempotent element Q satisfying $PQ = Q$, $QP = P$ for some projection $P \in \mathcal{N}$ is a Jordan all-derivable point of the nest subalgebra Alg$\mathcal{N}$ of $\mathcal{R}$.

Key words. Jordan derivations, Triangular rings, Nest algebras.

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