Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 22, pp. 979-999, September 2011

ELA
http://math.technion.ac.il/iic/ela

NEUTRAL SUBSPACES OF PAIRS OF SYMMETRIC/SKEWSYMMETRIC REAL MATRICES*

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Abstract. Let A and B be $n \times n$ real matrices with A symmetric and B skewsymmetric. Obviously, every simultaneously neutral subspace for the pair (A, B) is neutral for each Hermitian matrix X of the form $X = \mu A + i\lambda B$, where μ and λ are arbitrary real numbers. It is well-known that the dimension of each neutral subspace of X is at most $\mathrm{In}_+(X) + \mathrm{In}_0(X)$, and similarly, the dimension of each neutral subspace of X is at most $\mathrm{In}_-(X) + \mathrm{In}_0(X)$. These simple observations yield that the maximal possible dimension of an (A, B)-neutral subspace is no larger than

$$\min\{\min\{\mathrm{In}_+(\mu A + i\lambda B) + \mathrm{In}_0(\mu A + i\lambda B), \mathrm{In}_-(\mu A + i\lambda B) + \mathrm{In}_0(\mu A + i\lambda B)\}\},$$

where the outer minimum is taken over all pairs of real numbers (λ, μ). In this paper, it is proven that the maximal possible dimension of an (A, B)-neutral subspace actually coincides with the above expression.

Key words. Symmetric matrix, Skewsymmetric matrix, Hermitian matrix, Inertia, Neutral subspace.

AMS subject classifications. 15A21, 15A22, 15B57.

*Received by the editors on July 18, 2010. Accepted for publication on September 6, 2011. Handling Editor: Bryan L. Shader.

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