Electronic Journal of Linear Algebra ISSN 1081-3810
A publication of the International Linear Algebra Society
Volume 22, pp. 10-21, January 2011

ELA
http://math.technion.ac.il/iic/ela

MINIMUM SEMIDEFINITE RANK OF OUTERPLANAR GRAPHS AND THE TREE COVER NUMBER*

FRANCESCO BARIOLI†, SHAUN M. FALLAT‡, LON H. MITCHELL§, AND SIVARAM K. NARAYAN¶

Abstract. Let $G = (V, E)$ be a multigraph with no loops on the vertex set $V = \{1, 2, \ldots, n\}$. Define $S_+(G)$ as the set of symmetric positive semidefinite matrices $A = [a_{ij}]$ with $a_{ij} \neq 0$, $i \neq j$, if $ij \in E(G)$ is a single edge and $a_{ij} = 0$, $i \neq j$, if $ij \notin E(G)$. Let $M_+(G)$ denote the maximum multiplicity of zero as an eigenvalue of $A \in S_+(G)$ and $\mathrm{mr}_+(G) = |G| - M_+(G)$ denote the minimum semidefinite rank of G. The tree cover number of a multigraph G, denoted $T(G)$, is the minimum number of vertex disjoint simple trees occurring as induced subgraphs of G that cover all of the vertices of G. The authors present some results on this new graph parameter $T(G)$. In particular, they show that for any outerplanar multigraph G, $M_+(G) = T(G)$.

Key words. Minimum rank graph, Maximum multiplicity, Minimum semidefinite rank, Outerplanar graphs, Tree cover number.

AMS subject classifications. 05C50, 15A03, 15A18.

*Received by the editors on January 28, 2010. Accepted for publication on December 17, 2010. Handling Editor: Bryan L. Shader.

†Department of Mathematics, University of Tennessee at Chattanooga, Chattanooga TN, 37403 (francesco-barioli@utc.edu).

‡Department of Mathematics and Statistics, University of Regina, Regina, SK, Canada (sfallat@math.uregina.ca). Research supported in part by an NSERC Discovery research grant.

§Department of Mathematics and Applied Mathematics, Virginia Commonwealth University, Richmond, VA 23284-2014 (lmitchell2@vcu.edu).

¶Department of Mathematics, Central Michigan University, Mount Pleasant, MI 48859 (naray1sk@cmich.edu). Thanks to Central Michigan University for its support during his sabbatical leave in Fall of 2009 and MSRI for providing travel support to BIRS.