

## A SIMPLE CLOSED FORM FOR TRIANGULAR MATRIX POWERS\*

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**Abstract.** Given a  $k \times k$  triangular matrix  $M = [m_{i,j}]$  with unique diagonal elements, a simple recursive formula is used to define a set of  $\binom{k+2}{3}$  power factors,  $p_{i,j,s}$ , which are independent of the power to which the matrix is raised. Then for any power of  $M$ , negative, zero or positive (positive only, if the matrix is singular), the  $(i, j)$ -th element of  $M^n$  is given by a linear combination of power factors and powers of diagonal elements, namely  $m_{i,j} = \sum_{s=i}^j p_{i,j,s} m_{s,s}^{n-1}$ .

**Key words.** Matrix, Triangular, Powers, Closed form.

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