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FAST LOW RANK APPROXIMATIONS OF MATRICES AND TENSORS*

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Abstract. In many applications such as data compression, imaging or genomic data analysis, it is important to approximate a given $m \times n$ matrix A by a matrix B of rank at most k which is much smaller than m and n. The best rank k approximation can be determined via the singular value decomposition which, however, has prohibitively high computational complexity and storage requirements for very large m and n.

We present an optimal least squares algorithm for computing a rank k approximation to an $m \times n$ matrix A by reading only a limited number of rows and columns of A. The algorithm has complexity $\mathcal{O}(k^2 \max(m, n))$ and allows to iteratively improve given rank k approximations by reading additional rows and columns of A. We also show how this approach can be extended to tensors and present numerical results.

Key words. Singular value decomposition, CUR decomposition, Rank k approximation, Least squares, Tucker decomposition.

AMS subject classifications. 15A18, 15A69, 65F15, 93E24.

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