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THE SOLVABILITY CONDITIONS OF MATRIX EQUATIONS WITH K-INVOLUTION*

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Abstract. Let $m \times m$ complex matrix P and $n \times n$ complex matrix Q be k-involutions, i.e., $P^{k-1} = P^{-1}$, $Q^{k-1} = Q^{-1}$ for some integer $k \geq 2$. An $m \times n$ complex matrix A is (P, Q, β)-symmetric if $PAQ^{-1} = \lambda^{\beta} A$, or (P, Q, α, β)-symmetric if $PAQ^{-\alpha} = \lambda^{\beta} A$, where $\lambda = e^{2\pi i/k}$ and $\alpha, \beta \in \{1, 2, \ldots, k\}$. In this paper, for given matrices X, Y, E, F with appropriate sizes, the solvability of matrix equations $AX = E$ and $Y^*A = F$ under (P, Q, β)- and (P, Q, α, β)-constraints, respectively, are investigated. Meanwhile, the associated optimal approximation problem is also considered when the above P and Q are unitary.

Key words. k-Involution, (P, Q, β)-Symmetric matrices, (P, Q, α, β)-Symmetric matrices, Matrix equations, Optimal approximation.

AMS subject classifications. 65F05, 15A24.

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