

A NOTE ON MINIMUM RANK AND MAXIMUM NULLITY OF SIGN PATTERNS*

LESLIE HOGBEN[†]

Abstract. The minimum rank of a sign pattern matrix is defined to be the smallest possible rank over all real matrices having the given sign pattern. The maximum nullity of a sign pattern is the largest possible nullity over the same set of matrices, and is equal to the number of columns minus the minimum rank of the sign pattern. Definitions of various graph parameters that have been used to bound maximum nullity of a zero-nonzero pattern, including path cover number and edit distance, are extended to sign patterns, and the SNS number is introduced to usefully generalize the triangle number to sign patterns. It is shown that for tree sign patterns (that need not be combinatorially symmetric), minimum rank is equal to SNS number, and maximum nullity, path cover number and edit distance are equal, providing a method to compute minimum rank for tree sign patterns. The minimum rank of small sign patterns is determined.

Key words. Minimum rank, Maximum nullity, Sign pattern, Tree sign pattern, Asymmetric minimum rank, Path cover number, Edit distance, SNS sign pattern, SNS number, Ditree, Matrix.

AMS subject classifications. 15B35, 05C50, 15A03, 05C05, 94A05, 68Q17.

*Received by the editors on December 15, 2009. Accepted for publication on February 27, 2011. Handling Editor: Bryan Shader.

[†]Department of Mathematics, Iowa State University, Ames, IA 50011, USA (lhogben@iastate.edu), and American Institute of Mathematics, 360 Portage Ave, Palo Alto, CA 94306 (hogben@aimath.org).