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COMPUTING CODIMENSIONS AND GENERIC CANONICAL FORMS FOR GENERALIZED MATRIX PRODUCTS*

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Abstract. A *generalized matrix product* can be *formally* written as $A_p^{s_p} A_{p-1}^{s_{p-1}} \cdots A_2^{s_2} A_1^{s_1}$, where $s_i \in \{-1, +1\}$ and $(A_1, \ldots, A_p)$ is a tuple of (possibly rectangular) matrices of suitable dimensions. The *periodic eigenvalue problem* related to such a product represents a nontrivial extension of generalized eigenvalue and singular value problems. While the classification of generalized matrix products under eigenvalue-preserving similarity transformations and the corresponding canonical forms have been known since the 1970's, finding *generic* canonical forms has remained an open problem. In this paper, we aim at such generic forms by computing the codimension of the orbit generated by all similarity transformations of a given generalized matrix product. This can be reduced to computing the so called cointeractions between two different blocks in the canonical form. A number of techniques are applied to keep the number of possibilities for different types of cointeractions limited. Nevertheless, the matter remains highly technical; we therefore also provide a computer program for finding the codimension of a canonical form, based on the formulas developed in this paper. A few examples illustrate how our results can be used to determine the generic canonical form of least codimension. Moreover, we describe an algorithm and provide software for extracting the generically regular part of a generalized matrix product.

Key words. Matrix product, Periodic eigenvalue problem, Canonical form, Generic Kronecker structure, Cyclic quiver, Orthogonal reduction.

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