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## THE ALGEBRAIC RICCATI EQUATION WITH TOEPLITZ MATRICES AS COEFFICIENTS\*

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Abstract. It is shown that, under appropriate assumptions, the continuous algebraic Riccati equation with Toeplitz matrices as coefficients has Toeplitz-like solutions. Both infinite and sequences of finite Toeplitz matrices are considered, and also studied is the finite section method, which consists in approximating infinite systems by large finite truncations. The results are proved by translating the problem into  $C^*$ -algebraic language and by using theorems on the Riccati equation in general  $C^*$ -algebras. The paper may serve as another illustration of the usefulness of  $C^*$ -algebra techniques in matrix theory.

Key words. Algebraic Riccati equation, Toeplitz matrix,  $C^*$ -algebra.

AMS subject classifications. 47N70, 15A24, 15B05, 46L89, 47B35, 93C15.

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