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EVEN AND ODD TOURNAMENT MATRICES WITH MINIMUM RANK OVER FINITE FIELDS*

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Abstract. The $(0,1)$-matrix A of order n is a tournament matrix provided

$$A + A^T + I = J,$$

where I is the identity matrix, and $J = J_n$ is the all 1's matrix of order n. It was shown by de Caen and Michael that the rank of a tournament matrix A of order n over a field of characteristic p satisfies $\text{rank}_p(A) \geq (n-1)/2$ with equality if and only if n is odd and $AA^T = O$. This article shows that the rank of a tournament matrix A of even order n over a field of characteristic p satisfies $\text{rank}_p(A) \geq n/2$ with equality if and only if after simultaneous row and column permutations

$$AA^T = \begin{bmatrix} \pm J_m & O \\ O & O \end{bmatrix},$$

for a suitable integer m. The results and constructions for even order tournament matrices are related to and shed light on tournament matrices of odd order with minimum rank.

Key words. Tournament matrix, Rank.

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