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THE INVERSE EIGENVALUE AND INERTIA PROBLEMS FOR MINIMUM RANK TWO GRAPHS*

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Abstract. Let G be an undirected graph on n vertices and let $\mathcal{S}(G)$ be the set of all real symmetric $n \times n$ matrices whose nonzero off-diagonal entries occur in exactly the positions corresponding to the edges of G. Let $\mathrm{mr}(G)$ denote the minimum rank of all matrices in $\mathcal{S}(G)$, and $\mathrm{mr}_+(G)$ the minimum rank of all positive semidefinite matrices in $\mathcal{S}(G)$. All graphs G with $\mathrm{mr}(G) = 2$ and $\mathrm{mr}_+(G) = k$ are characterized; it is also noted that $\mathrm{mr}_+(G) = \alpha(G)$ for such graphs. This characterization solves the inverse inertia problem for graphs whose minimum rank is two. Furthermore, it is determined which diagonal entries are required to be zero, are required to be nonzero, or can be either for a rank minimizing matrix in $\mathcal{S}(G)$ when $\mathrm{mr}(G) = 2$. Collectively, these results lead to a solution to the inverse eigenvalue problem for rank minimizing matrices for graphs whose minimum rank is two.

Key words. Combinatorial matrix theory, Inertia, Inverse eigenvalue problem, Inverse inertia problem, Graph, Minimum positive semidefinite rank, Minimum rank, Nil vertex, Symmetric.

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