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LOEWNER MATRIX ORDERING IN ESTIMATION OF THE SMALLEST SINGULAR VALUE*

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Abstract. In this paper, some new lower bounds for the smallest singular value of a square complex matrix A are derived. A key tool to obtain these bounds is using some Hermitian matrices which are, in the sense of Loewner matrix ordering, below the Hermitian part of A or, more generally, below the Hermitian part of the unitary equivalences of A. Two types of new bounds are proposed. The first bound can be applied to matrices with positive diagonal entries and strictly diagonally dominant Hermitian parts. It is always at least as large (never worse) as pure Gersgorin-based bound due to C.R. Johnson [C.R. Johnson. A Gersgorin-type lower bound for the smallest singular value. Linear Algebra Appl., 112:1–7, 1989.]. The other bound is complementary with the first one and it can be effectively applied to matrices whose Hermitian parts are very far from diagonal dominance.

Key words. Singular values, Loewner ordering, Gudkov condition, Diagonal dominance.

AMS subject classifications. 65F15, 15A18, 15A42, 15A48.

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