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## AN EXTENSION OF THE PERTURBATION ANALYSIS FOR THE DRAZIN INVERSE\*

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Abstract. Let A denote a square complex matrix and let E be a perturbation matrix. The purpose of this paper is to investigate the perturbation of the Drazin inverse when B = A + E satisfies the rank conditions rank  $A^r = \operatorname{rank} B^s = \operatorname{rank} A^r B^s$ , where r and s denote the indices of A and B, respectively. We will derive an explicit representation of  $B^D$  as a function of A and  $B^k - A^j$ , for certain positive integers j, k. We emphasize that the matrix  $I + (A^D)^j (B^k - A^j)$  could be singular and the perturbation analysis will be carried out by using inner inverses. In addition, we exhibit inequalities bounding the errors  $||B^D - A^D||/||A^D||$  and  $||BB^D - AA^D||$ . Examples will be given which show that these bounds recover others given in the literature and can be significant to those cases which can not be bounded using the previous known results. Alternatively, we shall formulate analogous perturbation results for the perturbed matrix B such that rank  $A^r = \operatorname{rank} B^s = \operatorname{rank} B^s A^r$ .

Key words. Drazin inverse, Inner inverses, Projectors, Perturbation, Upper bound.

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