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PAIRS OF MATRICES, ONE OF WHICH COMMUTES WITH THEIR COMMUTATOR*

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Abstract. Let A, B be $n \times n$ complex matrices such that C = AB - BA and A commute. For n = 2, we prove that A, B are simultaneously triangularizable. For $n \ge 3$, we give an example of matrices A, B such that the pair (A, B) does not have property L of Motzkin-Taussky, and such that B and C are not simultaneously triangularizable. Finally, we estimate the complexity of the Alp'in-Koreshkov's algorithm that checks whether two matrices are simultaneously triangularizable. Practically, one cannot test a pair of numerical matrices of dimension greater than five.

Key words. Nilpotent matrix, Property L, Commutator, Quasi-commute.

AMS subject classifications. 15A27, 15A22.

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