

SPECTRA OF WEIGHTED ROOTED GRAPHS HAVING PRESCRIBED SUBGRAPHS AT SOME LEVELS*

OSCAR ROJO[†], MARIA ROBBIANO[‡], DOMINGOS M. CARDOSO[§], AND ENIDE A.
 MARTINS[§]

Abstract. Let \mathcal{B} be a weighted generalized Bethe tree of k levels ($k > 1$) in which n_j is the number of vertices at the level $k - j + 1$ ($1 \leq j \leq k$). Let $\Delta \subseteq \{1, 2, \dots, k - 1\}$ and $F = \{\mathcal{G}_j : j \in \Delta\}$, where \mathcal{G}_j is a prescribed weighted graph on each set of children of \mathcal{B} at the level $k - j + 1$. In this paper, the eigenvalues of a block symmetric tridiagonal matrix of order $n_1 + n_2 + \dots + n_k$ are characterized as the eigenvalues of symmetric tridiagonal matrices of order j , $1 \leq j \leq k$, easily constructed from the degrees of the vertices, the weights of the edges, and the eigenvalues of the matrices associated to the family of graphs F . These results are applied to characterize the eigenvalues of the Laplacian matrix, including their multiplicities, of the graph $\mathcal{B}(F)$ obtained from \mathcal{B} and all the graphs in $F = \{\mathcal{G}_j : j \in \Delta\}$; and also of the signless Laplacian and adjacency matrices whenever the graphs of the family F are regular.

Key words. Weighted graph, Laplacian matrix, Signless Laplacian matrix, Adjacency matrix, Generalized Bethe tree.

AMS subject classifications. 05C50, 15A18.

*Received by the editors on November 24, 2010. Accepted for publication on May 20, 2011.
 Handling Editor: Stephen J. Kirkland.

[†]Departamento de Matemáticas, Universidad Católica del Norte, Antofagasta, Chile (orojo@ucn.cl). Research supported by Project Fondecyt Regular 1100072, Chile.

[‡]Departamento de Matemáticas, Universidad Católica del Norte, Antofagasta, Chile (mrob-biano@ucn.cl). Research partially supported by Fondecyt - IC Project 11090211, Chile.

[§]Departamento de Matemática, Universidade de Aveiro, Aveiro, Portugal (dcardoso@ua.pt, enide@ua.pt). Research supported by the Centre for Research and Development in Mathematics and Applications from the Fundação para a Ciência e a Tecnologia - FCT (cofinanced by European Community Fund FEDER/POCI 2010) and project PTDC/MAT/112276/2009.