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LINEAR MAPS PRESERVING THE IDEMPOTENCY OF JORDAN PRODUCTS OF OPERATORS*

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Abstract. Let $\mathcal{B}(\mathcal{X})$ be the algebra of all bounded linear operators on a complex Banach space \mathcal{X} and let $\mathcal{I}^*(\mathcal{X})$ be the set of non-zero idempotent operators in $\mathcal{B}(\mathcal{X})$. A surjective map $\varphi : \mathcal{B}(\mathcal{X}) \to \mathcal{B}(\mathcal{X})$ preserves nonzero idempotency of the Jordan products of two operators if for every pair $A, B \in \mathcal{B}(\mathcal{X})$, the relation $AB + BA \in \mathcal{I}^*(\mathcal{X})$ implies $\varphi(A)\varphi(B) + \varphi(B)\varphi(A) \in \mathcal{I}^*(\mathcal{X})$. In this paper, the structures of linear surjective maps on $\mathcal{B}(\mathcal{X})$ preserving the nonzero idempotency of Jordan products of two operators are given.

Key words. Banach space, Preserver, Idempotent, Jordan product.

AMS subject classifications. 47B49.

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